Modeling of Planar Circuits Including the Effect of Space-Varying Surface Impedances

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Abstract—**The calculation of microstrip circuits including the effect of lumped impedances can be done by describing the lumped elements mathematically with the help of Δ-functions [1]. This approach proceeds on the assumption of impedances with infinite small extension in one dimension. This approach is generalized for impedances of finite extend. Therefore space-varying surface impedances are introduced that are incorporated into the mixed space-spectral domain analysis. The circuit is embedded in layered media and is fed by an arbitrary number of planar lines. Examples for microstrip lines with an absorbing impedance region are given.**

I. Model

We start with a multiport circuit that is embedded in a layered medium (Fig. 1). The total surface current on the metallic structure can be split into the current distribution on the circuit (represented by the sum over N basis functions) and the forward and backward travelling currents on the K feedlines:

$$\vec{J}_{\text{tot}}(x,y) = \vec{J}_c + \vec{J}_{\text{lines}} = \sum_n^N I_n \frac{f_n(x,y)}{b_n}\vec{u}_n + \sum_k^K \left(I_k^{\text{in}} \frac{f_k^{\text{in}}(x,y)}{b_k} + I_k^{\text{out}} \frac{f_k^{\text{out}}(x,y)}{b_k} \right)\vec{u}_k. \tag{1}$$

with b_n as the width of the nth current mode and b_k as the width of the kth line. The current on circuit $\vec{J}_c$ is described by N asymmetric piecewise sinusoidal basis functions $f_n(x, y)$. The number of basis functions depends on the complexity of the circuit that has to be analyzed (e.g., $N = 513$ for the Wilkinson coupler [1] shown in Fig. 1). The K feedlines are represented by semi-infinite homogeneous lines [2].

The electric field has to fulfill the surface impedance boundary condition on the circuit:

$$\vec{E}_{\text{tot}}(x,y)\,|_{\tan} = Z_{\text{tot}}(x,y)\,\vec{J}_{\text{tot}}(x,y). \tag{2}$$

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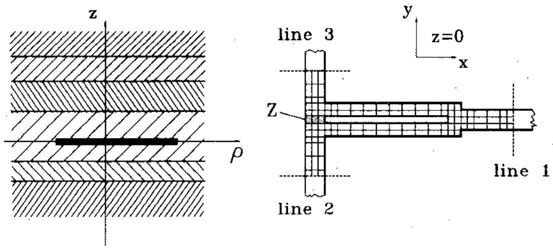

Fig. 1. Stratified structure with planar circuit (example).

The space-varying surface impedance consists of two parts

$$Z_{\text{tot}}(x,y) = \begin{cases} Z_c, & \text{on the homogeneous lines,} \\ Z_c + Z(x,y) & \text{elsewhere.} \end{cases} \tag{3}$$

Thus, in addition to a finite conductivity Z_c that represents the conductor losses, we have a surface impedance $Z(x, y)$ on the circuit (Z_c, $Z(x, y)$ complex values). By this space-varying surface impedance different kinds of metallisation can be modeled (e.g., a superconductive film) or/and we can model impedances Z^{eff} of finite size. In order to formulate an integral equation we define the Green's function $\overleftrightarrow{G}(\vec{r}, \vec{r}')$ of the stratified medium [3] with

$$\vec{E}_{\text{tot}}(x,y,z) = \int_{x'}\int_{y'}\int_{z'} \overleftrightarrow{G}(x,y,z,x',y',z') \cdot \vec{J}_{\text{tot}}(x',y',z')\,dv'. \tag{4}$$

The right side of (4) is written as a twofold Fourier transform of its spectral components and inserted in (2). The integral equation is solved by the method of moments (see [1] for detailed information). Applying a modified method of Galerkin we obtain the following set of linear equations:

$$\sum_i^{N+K} I_i\left(Z_{ji} - Z_{ji}^{xy}\right) = \sum_k^K I_k^{\text{in}} V_{jk}, \quad \text{with } j = 1,\cdots,N+K, \tag{5}$$

$$Z_{ji} = \frac{1}{4\pi^2}\int_{k_x}\int_{k_y} \vec{u}_j\left(\overleftrightarrow{G}(k_x,k_y) - Z_c\overleftrightarrow{I}\right) \cdot \vec{u}_i \frac{F_i(k_x,k_y)F_j^*(k_x,k_y)}{b_i b_j}\,dk_x\,dk_y, \tag{6}$$

$$Z_{ji}^{xy} = \int_x\int_y \vec{u}_j\vec{u}_i Z(x,y)\frac{f_i(x,y)f_j(x,y)}{b_i b_j}\,dx\,dy, \tag{7}$$

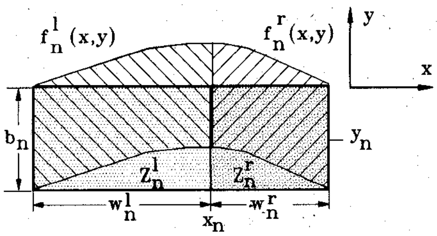

Fig. 2. Splitting of the bases function and the surface impedance into a left and a right term (x-mode).

$$V_{jk} = -\frac{1}{4\pi^2}\int_{k_x}\int_{k_y}\vec{u}_j\left(\overleftrightarrow{G}(k_x, k_y) - Z_c\overleftrightarrow{I}\right) \cdot \vec{u}_k \frac{F_k^{\text{in}}(k_x, k_y)F_j^*(k_x, k_y)}{b_k b_j}\, dk_x\, dk_y. \quad (8)$$

($F_n(k_x, k_y)$ is the Fourier transform of the nth basis function $f_n(x, y)$ and $\overleftrightarrow{G}(k_x, k_y)$ is the Fourier transform of the Green's function $\overleftrightarrow{G}(\vec{r}, \vec{r}')$.)

II. Space-Varying Impedances

The rectangular net used for the modeling of the current is *simultaneously* used for the discretisation of the space-varying surface impedance $Z(x, y)$. On each rectangular area defined by this net the surface impedance is set to be constant. In order to involve this approach with the theory applied so far, the basis function $f_n(x, y)$ (e.g., for a current in x-direction) is split up into a *left* and a *right* term (Fig. 2), and we get

$$\begin{aligned} Z(x, y)\vec{J}_c(x, y) &= \sum_n^N I_n\big(A_n^l(x, y) + A_n^r(x, y)\big)\frac{f_n(x, y)}{b_n}Z(x, y)\vec{u}_n \\ &= \sum_n^N I_n \frac{f_n^l(x, y) + f_n^r(x, y)}{b_n} \cdot Z(x, y)\vec{u}_n. \end{aligned} \quad (9)$$

The functions $A_n^{l/r}$ are defined as

$$A_n^l(x, y) = \begin{cases} 1, & \text{for } x_n - w_n^l < x < x_n; \\ & y_n - b_n/2 < y < y_n + b_n/2, \\ 0, & \text{elsewhere}, \end{cases} \quad (10a)$$

and

$$A_n^r(x, y) = \begin{cases} 1, & \text{for } x_n < x < x_n + w_n^r; \\ & y_n - b_n/2 < y < y_n + b_n/2, \\ 0, & \text{elsewhere}. \end{cases} \quad (10b)$$

With this we have piecewise constant surface impedances

$$Z_n(x, y) = Z_n^l A_n^l(x, y) + Z_n^r A_n^r(x, y)$$

with

$$Z_n^l, Z_n^r \text{ const.} \quad (11)$$

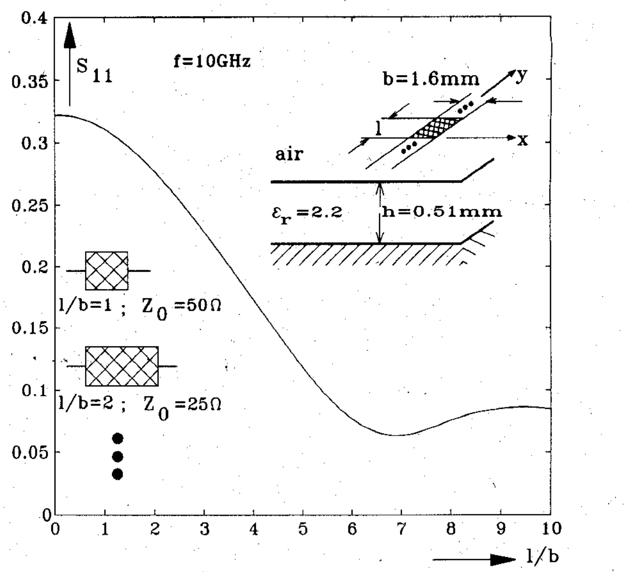

Fig. 3. Reflection of an impedance region ($Z^{\text{eff}} = 50\,\Omega$; surface impedance Z_0).

We obtain finally

$$Z(x, y)\vec{J}_c(x, y) = \sum_n^N I_n \frac{f_n^l(x, y)Z_n^l + f_n^r(x, y)Z_n^r}{b_n}\vec{u}_n. \quad (12)$$

Because of the simple basis functions, (7) can be evaluated analytically now by

$$Z_{ji}^{xy} = \frac{\vec{u}_j\vec{u}_i}{b_i b_j}\left(Z_i^l\int_x\int_y f_i^l(x, y)f_j(x, y)\,dx\,dy + Z_i^r\int_x\int_y f_i^r(x, y)f_j(x, y)\,dx\,dy\right). \quad (13)$$

The effective impedance of the region of the n'th basis function is given by

$$Z_n^{\text{eff}} = Z_n^l\frac{w_n^l}{b_n} + Z_n^r\frac{w_n^r}{b_n}. \quad (14)$$

The description for currents in y-directions is done in an analogous way.

III. Examples

In the first example the length l of a region with constant surface impedance on a microstrip line ($Z_c = 0$) is varied (Fig. 3). For a constant impedance value Z^{eff} of the complete region

$$Z^{\text{eff}} = Z_0\frac{l}{b}$$

with

$$Z(x, y) = \begin{cases} Z_0, & \text{for } -b/2 < y < b/2; \quad 0 < x < l, \\ 0, & \text{else}, \end{cases} \quad (15)$$

the value of the surface impedance Z_0 must decrease for increasing length of the region ($l = 0$ is the lumped element).

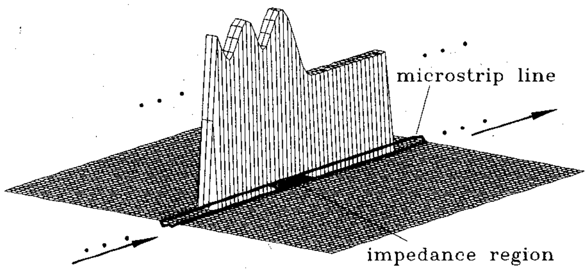

Fig. 4. Amplitude of the current on a microstrip line with an impedance region of finite extend ($l/b = 5$; for data see Fig. 3).

Fig. 3 shows the reflection coefficient S_{11} as a function of the length l. As was to be expected, the reflection decreases with increasing length of the impedance region (for $l/b = \infty$ we would expect $S_{11} = 0$). The current distribution for $l/b = 5$ is shown in Fig. 4.

Surface varying impedances can be used for matched loads. Suitable results are obtained applying an exponential increase of the surface impedance. As an example Fig. 5 shows the current distribution for a microstrip line terminated with such a load.

IV. Conclusion

Space-varying impedances allow the calculation of planar circuits with finite areas of different surface impedances. This often gives a more realistic model than the assumption of lumped elements which are defined by Δ functions. Furthermore the approach outlined here can advantageously be used for planar circuits with areas of different conductivities as it is, e.g., with the application of superconductivity.

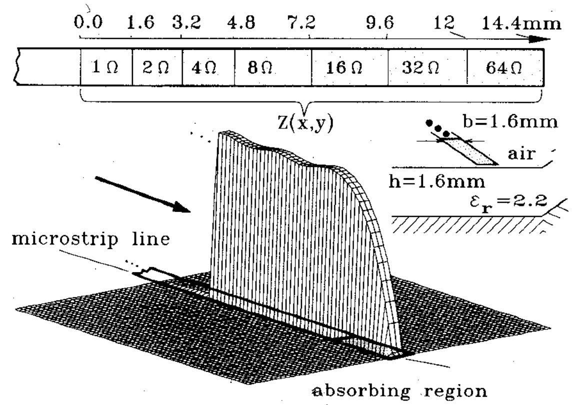

Fig. 5. Amplitude of the current on a microstrip line terminated by an absorber (f = 10 GHz).

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